Foreword to the First Edition

In the summer of 2003 we began designing multi-track recording and mixing software – Orinj at RecordingBlogs.com – a software application that will take digitally recorded audio tracks and will mix them into a complete song with all the needed audio production effects. Manipulating digital sound, as it turned out, was not easy. We had to find the answers of many questions, including what digital audio was, how we could mix audio tracks, how we could track the amplitude of digital sound so that we could apply compression, how we could track frequencies so that we could equalize, what a good model of artificial reverb would be, and many others. Bits of relevant information were available, albeit not always well organized and not always intuitive.

"Digital Signal Processing for Audio Applications" provides much of the needed information. It is a simple structured approach to understanding how digitally recorded sound can be manipulated. It presents and explains, and sometimes derives, the mathematical theory that the DSP user can employ in designing sound manipulating applications.

Although this book introduces much mathematics, we have designed it not for mathematicians, but for the engineers and hobbyists, who would be interested in the practical applications of DSP and not in its theoretical derivations. If properly explained, much of the practical DSP applications reduce to simple algebra. This said, we have included a sufficient amount of theory to provide an explanation of why DSP works the way it does. It is important for practitioners to have a good understanding of how DSP concepts come about. Much of the available DSP information has too much theory and not enough examples. Much of it has too many practical examples and not enough theoretical backing. We hope to have found the proper balance.

We hope you enjoy this book and make use of its definitions, explanations, and numerous examples.

The author and the administrators of www.recordingblogs.com
Foreword to the Third Edition

This edition contains Java code samples for several digital signal processing effects – delay, chorus, equalizer, reverb, compressor, wah wah, pitch shift, and more. These are a significant addition and are presented in a separate volume 2. Selected relevant sections of the previous edition of this book are also placed in volume 2.

The first edition of this book focused on signal frequencies – identifying them, filtering them out, changing their magnitude, and so on. This is a huge part of DSP for audio, but there is more. The second edition introduced significant additions: wavelet transforms and data compression, more windows, and elliptic filters. This third edition includes shelving and peak filters, improves the discussion of the Hilbert transform, and, of course, introduces a number of code samples as part of volume 2.

We hope you enjoy this edition.

The author and the administrators of www.recordingblogs.com
# Table of Contents

Chapter 1. Introduction ........................................................................................................................... 20

Chapter 2. Simple waves in continuous time........................................................................................ 21
  2.1. Initial phase .................................................................................................................................... 21
  2.2. Peak amplitude ............................................................................................................................... 23
  2.3. Frequency ....................................................................................................................................... 24

Chapter 3. Simple waves in discrete time.............................................................................................. 27
  3.1. Sampling ......................................................................................................................................... 27
  3.2. Discrete simple waves ................................................................................................................... 28
  3.3. The Nyquist-Shannon sampling theorem .................................................................................. 29

Chapter 4. Complex signals and simple DSP operations.................................................................... 31
  4.1. Mathematical representation of the complex signal ................................................................... 32
  4.2. Mixing and simple delays ............................................................................................................. 33
  4.3. A running average filter ................................................................................................................ 33
  4.4. Frequencies after the running average filter .............................................................................. 35
  4.5. A better way to measure amplitude ............................................................................................ 38

Chapter 5. Introduction to Fourier analysis .......................................................................................... 42
  5.1. Orthogonal functions and the Fourier series ............................................................................ 42
  5.2. Frequency content of signals ....................................................................................................... 44
  5.3. Computing the content of an example complex signal ........................................................... 46

Chapter 6. Distortion ............................................................................................................................... 49
  6.1. Distortion ....................................................................................................................................... 49
  6.2. Upsampling and downsampling .................................................................................................. 54

Chapter 7. A good low pass filter .......................................................................................................... 56
  7.1. Convolution of simple waves ...................................................................................................... 56
  7.2. An ideal low pass filter ................................................................................................................. 58
  7.3. Design of the low pass filter ........................................................................................................ 59
  7.4. Alternative form of the low pass filter ....................................................................................... 63

Chapter 8. Properties of the low pass filter ......................................................................................... 64
  8.1. Magnitude response ...................................................................................................................... 64
  8.2. Impulse response ........................................................................................................................... 68
  8.3. Phase response ............................................................................................................................... 71

Chapter 9. All pass, high pass, band pass, and band stop filters........................................................ 76
  9.1. Incorrect finite impulse response high pass filter ..................................................................... 76
  9.2. Finite impulse response all pass filter ......................................................................................... 77
  9.3. Finite impulse response high pass filter ..................................................................................... 78
<table>
<thead>
<tr>
<th>Chapter 9. Finite impulse response band pass and band stop filters</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.4. Finite impulse response band pass and band stop filters</td>
<td>79</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Chapter 10. Windowing of finite impulse response filters</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.1. Rectangular window</td>
<td>82</td>
</tr>
<tr>
<td>10.2. Hamming window</td>
<td>82</td>
</tr>
<tr>
<td>10.3. Bartlett-Hann window</td>
<td>84</td>
</tr>
<tr>
<td>10.4. Tukey window</td>
<td>85</td>
</tr>
<tr>
<td>10.5. Kaiser window</td>
<td>86</td>
</tr>
<tr>
<td>10.6. Windowing of high pass filters</td>
<td>88</td>
</tr>
<tr>
<td>10.7. Windowed band pass filters and band stop filters</td>
<td>89</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Chapter 11. Simple equalizer</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chapter 12. The continuous and discrete Fourier transforms</td>
<td>Page</td>
</tr>
<tr>
<td>12.1. Definitions</td>
<td>95</td>
</tr>
<tr>
<td>12.2. A simple DFT example</td>
<td>96</td>
</tr>
<tr>
<td>12.3. The DFT and FIR filters</td>
<td>100</td>
</tr>
<tr>
<td>12.4. Using the inverse DFT</td>
<td>103</td>
</tr>
<tr>
<td>12.5. Sampling of the continuous inverse Fourier transform</td>
<td>107</td>
</tr>
<tr>
<td>12.6. Deriving the family of Hamming windows</td>
<td>109</td>
</tr>
<tr>
<td>12.7. Dolph-Chebychev window</td>
<td>111</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Chapter 13. Pitch shifting</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.1. Pitch shifting with the discrete Fourier transform</td>
<td>119</td>
</tr>
<tr>
<td>13.2. A note on the fast Fourier transform</td>
<td>124</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Chapter 14. Spectral analysis and window measures</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>14.1. Coherent gain</td>
<td>128</td>
</tr>
<tr>
<td>14.2. Equivalent noise bandwidth</td>
<td>129</td>
</tr>
<tr>
<td>14.3. Processing loss</td>
<td>132</td>
</tr>
<tr>
<td>14.4. Scalloping loss</td>
<td>133</td>
</tr>
<tr>
<td>14.5. Worst-case processing loss</td>
<td>135</td>
</tr>
<tr>
<td>14.6. Sidelobe falloff</td>
<td>136</td>
</tr>
<tr>
<td>14.7. Overlap correlation and amplitude flatness</td>
<td>137</td>
</tr>
<tr>
<td>14.8. Other window measures</td>
<td>140</td>
</tr>
<tr>
<td>14.9. Window performance</td>
<td>142</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Chapter 15. Comb filters and reverberation</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>15.1. Feedforward comb filter</td>
<td>147</td>
</tr>
<tr>
<td>15.2. Feedback comb filter</td>
<td>148</td>
</tr>
<tr>
<td>15.3. Comb filters and the Shroeder reverb</td>
<td>149</td>
</tr>
<tr>
<td>15.4. An alternative reverb implementation</td>
<td>154</td>
</tr>
<tr>
<td>15.5. Impulse based reverb through de-convolution</td>
<td>155</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Chapter 16. Z transforms, the discrete-time Fourier transform, and transfer functions</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>16.1. Definitions</td>
<td>166</td>
</tr>
</tbody>
</table>
# Table of contents

16.1. Transfer functions of comb filters ................................................................. 169  
16.2. Transfer functions of FIR filters ................................................................. 171  
16.3. Transfer functions of IIR filters ................................................................. 174  

Chapter 17. The Laplace transform and Butterworth filters ........................................ 176  
17.1. Transfer functions using the Laplace transform ........................................ 177  
17.2. An example of a Butterworth low pass filter ............................................. 178  
17.3. Motivation behind the Butterworth low pass filter .................................... 181  

Chapter 18. The bilinear transformation and IIR filter transformations ..................... 187  
18.1. The bilinear transformation ......................................................................... 187  
18.2. Butterworth filters with the bilinear transformation .................................. 188  
18.3. High pass and other Butterworth filters ..................................................... 189  
18.4. Higher order Butterworth filters ................................................................. 193  
18.5. Phase response of the Butterworth filter .................................................... 194  
18.6. Equivalent noise bandwidth of the Butterworth filter ............................... 195  
18.7. Warping of the frequency domain and the biquad transformation ............ 197  

Chapter 19. Other IIR filter prototypes ................................................................. 199  
19.1. The Bessel filter ............................................................................................... 199  
19.2. The Chebychev type I filter ......................................................................... 203  
19.3. The Chebychev type II filter ........................................................................ 208  
19.4. A prototype notch filter ................................................................................ 215  
19.5. Shelving and peak filters .............................................................................. 217  

Chapter 20. Equiripple filters ................................................................................ 223  
20.1. Elliptic filter .................................................................................................... 223  
20.2. An equiripple FIR filter ................................................................................ 229  

Chapter 21. Designing filters through optimization ................................................ 235  

Chapter 22. IIR filters, FIR filters, quantization, and dithering ............................... 241  
22.1. Dithering ....................................................................................................... 244  
22.2. An example of noise shaping ....................................................................... 246  

Chapter 23. Dynamic compression ....................................................................... 248  
23.1. Detecting amplitude envelopes and the Hilbert transform ....................... 248  

Chapter 24. Data compression ............................................................................... 253  
24.1. Haar wavelet transform ............................................................................... 255  
24.2. Data compression with the Haar wavelet transform .................................. 260  
24.3. Daubechies Daub4 wavelet transform ....................................................... 262  
24.4. Data compression with the Daub4 wavelet transform .............................. 264  
24.5. Motivation for the wavelet transforms ......................................................... 265  

Appendix A. Windows ............................................................................................ 269
Appendix B. An IIR-FIR filter prototype ................................................................. 283
Appendix C. Appropriate filter length ........................................................................ 286
Table of Figures

Figure 1. A simple cosine wave ................................................................. 21
Figure 2. Simple waves with different initial phase .................................. 22
Figure 3. Two waves with inverted phase ................................................ 23
Figure 4. Waves with different phase and amplitude ............................. 24
Figure 5. Waves with different amplitude and frequency ....................... 25
Figure 6. A sampled simple wave ............................................................ 27
Figure 7. An example of aliasing ............................................................... 30
Figure 8. Frequencies in an acoustic guitar recording ............................ 31
Figure 9. A signal composed of three simple waves ............................... 31
Figure 10. A simple wave with cycle shorter than the length of the averaging operation ........................................ 35
Figure 11. Peak amplitude of frequencies after the averaging operation ........ 36
Figure 12. Peak amplitudes at selected frequencies ................................. 36
Figure 13. Peak amplitudes after three running average filters ............... 37
Figure 14. RMS amplitude after three different running average filters .......... 39
Figure 15. Magnitude response of three running average filters – RMS amplitude in decibels ... 41
Figure 16. Content of an example signal .................................................. 46
Figure 17. A closer look around 25 Hz ..................................................... 47
Figure 18. Hard clip of a simple wave ...................................................... 49
Figure 19. Frequency content of the original signal ................................. 50
Figure 20. Frequency content after the hard clip ...................................... 50
Figure 21. Cubic soft clipper ................................................................. 51
Figure 22. The impact of odd order harmonics ....................................... 52
Figure 23. The impact of even order harmonics ..................................... 52
Figure 24. Frequency content of an expansion ........................................ 53
Figure 25. An example of upsampling ..................................................... 55
Figure 26. Magnitude response of an ideal low pass filter ....................... 59
Figure 27. Example partial filter ............................................................ 62
Figure 28. Magnitude response of the low pass filter ............................. 62
Figure 29. Comparison of two low pass filters ....................................... 63
Figure 30. Magnitude response of a typical filter .................................... 65
Figure 31. Magnitude response of three filters with different length ........ 67
Figure 32. Gibbs phenomenon in the pass band .................................... 68
Figure 33. Impulse response of the filter ............................................... 69
Figure 34. Impulse response of an example filter .................................... 70
Figure 35. Phase response of the example filter ..................................... 74
Figure 36. Phase response adjusted for periodicity ................................ 75
Figure 37. An incorrect high pass filter ................................................... 77
Figure 38. A good high pass filter .......................................................... 78
Figure 39. An example band pass filter ................................................ 80
Table of figures

Figure 40. Hamming window ................................................................................................................. 83
Figure 41. Impulse response of a low pass filter with the Hamming window ................................ 83
Figure 42. Magnitude responses of low pass filters with the Hamming and rectangular windows ..................................................................................................................................................................... 84
Figure 43. Magnitude response of low pass filters with and without the Bartlett-Hann window 85
Figure 44. Tukey window at three different $\alpha$ ...................................................................................... 86
Figure 45. Magnitude response of the Tukey window ........................................................................ 86
Figure 46. Kaiser window at three different $\alpha$ ..................................................................................... 88
Figure 47. Magnitude response of the Kaiser window ..................................................................... 88
Figure 48. Magnitude response of an equalizer .................................................................................... 91
Figure 49. Magnitude response of two filters ...................................................................................... 92
Figure 50. Magnitude response of an equalizer .................................................................................... 93
Figure 51. DFT magnitude of a simple wave ....................................................................................... 99
Figure 52. Magnitude response with the DFT ................................................................................... 101
Figure 53. Shifting and adding to the inverse DFT filter ..................................................................... 106
Figure 54. Magnitude response of an inverse DFT filter computed with the DFT ..................... 106
Figure 55. A more precise computation of the magnitude response of an inverse DFT filter... 107
Figure 56. A revised desired magnitude response ............................................................................. 110
Figure 57. The Dolph-Chebychev magnitude response .................................................................... 113
Figure 58. Dolph-Chebychev window ................................................................................................ 114
Figure 59. Magnitude response of a filter with the Dolph-Chebychev window ............................ 114
Figure 60. Dolph-Chebychev windows at three different $\omega_0$ .................................................. 116
Figure 61. Magnitude response of low pass filters with Dolph-Chebychev windows with different $\omega_0$ .............................................................................................................................................. 116
Figure 62. Dolph-Chebychev window with two different $L$ ............................................................. 117
Figure 63. Magnitude responses of low pass filters with the Dolph-Chebychev window at different $L$ ................................................................................................................................................ 117
Figure 64. Subsequent DFT segment discontinuity .......................................................................... 122
Figure 65. Cross-fading to remove discontinuities ............................................................................ 123
Figure 66. Magnitude content of a signal after the Fourier transform ........................................... 126
Figure 67. Magnitude content of the second signal after the Fourier transform ........................ 127
Figure 68. Magnitude content of the signal before and after the Hann window ........................ 128
Figure 69. Spectral leakage when zooming into the discrete Fourier transform ......................... 129
Figure 70. A visualization of a practical low pass filter and an ideal one ....................................... 130
Figure 71. A scallop from the discrete Fourier transform ............................................................... 134
Figure 72. Scalloping loss of the rectangular window .................................................................... 134
Figure 73. Scalloping loss of the Hann window .............................................................................. 135
Figure 74. Fourier transform of the rectangular window ................................................................. 136
Figure 75. Fourier transform of the rectangular window – zoom around the center ................. 137
Figure 76. Non-overlapping Hann windows ..................................................................................... 138
Figure 77. Total weight of six overlapping Hann windows .............................................................. 139
Figure 78. Amplitude flatness and overlap correlation of the Hann window ............................... 140
Figure 79. Fourier transform of the Gaussian window................................................................. 141
Figure 80. Fourier transform of the Dolph-Chebychev window .................................................... 141
Figure 81. A point where the stop band may start ............................................................................ 143
Figure 82. Performance measures for the Tukey window................................................................. 143
Figure 83. Another start of the stop band .......................................................................................... 144
Figure 84. Performance measures for the Blackman-Harris window .............................................. 144
Figure 85. Impulse response of a feedforward comb filter .............................................................. 148
Figure 86. Magnitude response of a feedforward comb filter ........................................................ 148
Figure 87. Magnitude response of a feedback comb filter .............................................................. 149
Figure 88. Reflections of the reverb ..................................................................................................... 151
Figure 89. Shroeder reverb .................................................................................................................... 152
Figure 90. Impulse response of an example Shroeder all pass filter ............................................... 152
Figure 91. Impulse response of an example Shroeder reverb ........................................................... 153
Figure 92. Impulse response of the Shroeder-Moore filter .............................................................. 155
Figure 93. Impulse response of a tapped delay line ........................................................................... 156
Figure 94. Impulse response of the first all pass filter ...................................................................... 157
Figure 95. Impulse response of the two all pass filters ..................................................................... 158
Figure 96. Impulse response after the two all pass filters and the comb filter .................................... 158
Figure 97. Impulse response of the reverb ............................................................................................ 159
Figure 98. A drum hit signal .................................................................................................................. 160
Figure 99. Convolving the drum hit with the impulse response of the reverb .................................... 160
Figure 100. A reverberated drum hit ................................................................................................... 160
Figure 101. A drum hit after a tapped delay line with two taps ......................................................... 161
Figure 102. A drum hit after a delayed Shroeder all pass filter ......................................................... 161
Figure 103. A drum hit after a delay and two Shroeder all pass filters .............................................. 162
Figure 104. A drum hit after two all pass filters and a feedforward comb filter ............................... 162
Figure 105. Impulse response of a reverb after the deterministic de-convolution ............................ 164
Figure 106. Impulse response of a reverb computed with optimization .......................................... 164
Figure 107. Magnitude response of a feedforward comb filter ........................................................ 170
Figure 108. Position of the ripples of the FIR filter ........................................................................... 173
Figure 109. Magnitude response of the Butterworth filter ............................................................... 180
Figure 110. Butterworth magnitude response function ................................................................. 181
Figure 111. Potential poles of the second order low pass Butterworth filter transfer function ......... 185
Figure 112. Bilinear and impulse invariant Butterworth filters ....................................................... 189
Figure 113. Magnitude response of an example high pass Butterworth filter .................................. 191
Figure 114. Magnitude response of an example band pass Butterworth filter ............................... 193
Figure 115. Magnitude response of Butterworth filters of different order ....................................... 194
Figure 116. Phase response of a Butterworth filter ......................................................................... 195
Figure 117. Magnitude response of a third order high pass Bessel filter ........................................ 201
Figure 118. Magnitude response of a second order band stop Bessel filter ....................................... 202
Table of figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>119</td>
<td>Phase response of the second order Butterworth and Bessel filters</td>
<td>203</td>
</tr>
<tr>
<td>120</td>
<td>Magnitude response of a second order low pass Chebychev type I filter</td>
<td>205</td>
</tr>
<tr>
<td>121</td>
<td>Magnitude of the Chebychev type I filter for three $\epsilon$</td>
<td>206</td>
</tr>
<tr>
<td>122</td>
<td>Magnitude response of the Chebychev type I filter for three $\epsilon$</td>
<td>206</td>
</tr>
<tr>
<td>123</td>
<td>Magnitude response of the second order high pass Chebychev type I filter</td>
<td>207</td>
</tr>
<tr>
<td>124</td>
<td>Chebychev type I filters of different order</td>
<td>208</td>
</tr>
<tr>
<td>125</td>
<td>Magnitude response of the Chebychev type II filter</td>
<td>211</td>
</tr>
<tr>
<td>126</td>
<td>Magnitude response of the Chebychev type II filter for three $\epsilon$</td>
<td>212</td>
</tr>
<tr>
<td>127</td>
<td>Magnitude response of a Chebychev type II band stop filter</td>
<td>213</td>
</tr>
<tr>
<td>128</td>
<td>Magnitude response of a second order and a fourth order Chebychev type II filters</td>
<td>214</td>
</tr>
<tr>
<td>129</td>
<td>Magnitude response of a notch filter</td>
<td>217</td>
</tr>
<tr>
<td>130</td>
<td>Magnitude response of a low-boost shelving filter</td>
<td>219</td>
</tr>
<tr>
<td>131</td>
<td>Magnitude response of a band-boost shelving filter</td>
<td>221</td>
</tr>
<tr>
<td>132</td>
<td>Magnitude response of a narrow band-boost shelving filter</td>
<td>221</td>
</tr>
<tr>
<td>133</td>
<td>Magnitude response of a peak filter</td>
<td>222</td>
</tr>
<tr>
<td>134</td>
<td>Gain functions of the elliptic filters of order 2 and 4</td>
<td>224</td>
</tr>
<tr>
<td>135</td>
<td>Roots of the transfer function of the fourth order high pass elliptic filter</td>
<td>228</td>
</tr>
<tr>
<td>136</td>
<td>Magnitude response of an example high pass elliptic filter of order 4</td>
<td>229</td>
</tr>
<tr>
<td>137</td>
<td>Magnitude response of the inverse DFT filter</td>
<td>230</td>
</tr>
<tr>
<td>138</td>
<td>Magnitude response after one and after 5 iterations</td>
<td>231</td>
</tr>
<tr>
<td>139</td>
<td>Magnitude response after many iterations</td>
<td>232</td>
</tr>
<tr>
<td>140</td>
<td>Magnitude response of an IIR filter computed with optimization</td>
<td>239</td>
</tr>
<tr>
<td>141</td>
<td>Two Butterworth filters with the same cutoff frequency</td>
<td>242</td>
</tr>
<tr>
<td>142</td>
<td>Example quantization errors</td>
<td>244</td>
</tr>
<tr>
<td>143</td>
<td>A dithered wave</td>
<td>245</td>
</tr>
<tr>
<td>144</td>
<td>Hilbert transform FIR filter</td>
<td>250</td>
</tr>
<tr>
<td>145</td>
<td>100 Hz after the Hilbert transform filter</td>
<td>251</td>
</tr>
<tr>
<td>146</td>
<td>Magnitude response of the Hilbert transform filter at low frequencies</td>
<td>251</td>
</tr>
<tr>
<td>147</td>
<td>An amplitude envelope computed after the Hilbert transform filter</td>
<td>252</td>
</tr>
<tr>
<td>148</td>
<td>An example signal to be compressed</td>
<td>253</td>
</tr>
<tr>
<td>149</td>
<td>Trend of the example signal</td>
<td>254</td>
</tr>
<tr>
<td>150</td>
<td>Trend of the level 2 Haar wavelet transform of the example signal</td>
<td>255</td>
</tr>
<tr>
<td>151</td>
<td>The level $N$ Haar wavelet</td>
<td>257</td>
</tr>
<tr>
<td>152</td>
<td>The level $N-1$ Haar wavelets</td>
<td>257</td>
</tr>
<tr>
<td>153</td>
<td>The level $N-2$ Haar wavelet</td>
<td>258</td>
</tr>
<tr>
<td>154</td>
<td>2:1 data compression with the Haar wavelet transform</td>
<td>260</td>
</tr>
<tr>
<td>155</td>
<td>Frequencies in the original signals</td>
<td>261</td>
</tr>
<tr>
<td>156</td>
<td>Frequencies in the compressed signal</td>
<td>261</td>
</tr>
<tr>
<td>157</td>
<td>5:1 compression with the Haar wavelet transform</td>
<td>265</td>
</tr>
<tr>
<td>158</td>
<td>5:1 compression with the Daubechies Daub4 wavelet transform</td>
<td>265</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>159</td>
<td>An example signal</td>
<td>266</td>
</tr>
<tr>
<td>160</td>
<td>Frequency content of the trend signal</td>
<td>267</td>
</tr>
<tr>
<td>161</td>
<td>Frequency content of the fluctuation signal</td>
<td>267</td>
</tr>
<tr>
<td>162</td>
<td>Magnitude response of a low pass filter with various windows</td>
<td>275</td>
</tr>
<tr>
<td>163</td>
<td>Magnitude response of a low pass filter with various window functions in the pass band</td>
<td>277</td>
</tr>
<tr>
<td>164</td>
<td>Window measures</td>
<td>279</td>
</tr>
<tr>
<td>165</td>
<td>Convolving an IIR and a FIR filter</td>
<td>284</td>
</tr>
<tr>
<td>166</td>
<td>Magnitude response of an under-sampled low pass filter in the original pass band</td>
<td>285</td>
</tr>
<tr>
<td>167</td>
<td>Main lobe of a FIR filter</td>
<td>287</td>
</tr>
<tr>
<td>168</td>
<td>Magnitude response of two filters of different length</td>
<td>288</td>
</tr>
</tbody>
</table>
Table of Equations

2.5. Simple sound wave in continuous time ................................................................. 6
3.2. Simple sound wave in discrete time .................................................................. 9
4.1. Complex signal in continuous time .................................................................. 13
4.2. Complex signal in discrete time ....................................................................... 13
4.3. Mixing .................................................................................................................. 14
4.4. Simple delay ........................................................................................................ 14
4.5. Running average filter ..................................................................................... 15
4.9. Root mean square (RMS) amplitude ............................................................... 19
4.11. Decibel ............................................................................................................... 21
5.2. Decomposition of a simple waves to remove phase ........................................ 23
5.3. Orthogonal simple waves ............................................................................... 23
5.15. Frequency content and amplitude ............................................................... 25
5.16. Frequency content and phase ......................................................................... 26
5.19. Fourier series .................................................................................................. 26
5.22. RMS amplitude of a simple wave ................................................................. 29
6.1. Hard clip distortion .......................................................................................... 30
6.2. Cubic soft clipper distortion ............................................................................ 32
6.3. Soft distortion ................................................................................................... 32
6.4.-6.5. Even order harmonics in distortion ......................................................... 34
6.7. Upsampling ......................................................................................................... 35
7.3. Convolution in continuous time ......................................................................... 37
7.4. Convolution in discrete time ............................................................................. 37
7.6. Convolution of simple waves in continuous time ........................................... 38
7.8. Convolution of simple waves in discrete time ................................................ 38
7.11. Dirichlet kernel ................................................................................................ 41
7.15. Low pass filter derived with discrete computations ..................................... 42
8.5. Magnitude response of a finite impulse response filter .................................. 47
8.6. Kronecker delta function .................................................................................. 50
8.8. Impulse response of a finite impulse response filter ........................................ 50
8.10. General formula for a discrete-time filter ....................................................... 52
9.1. Finite impulse response all pass filter ............................................................. 58
9.2. Distributive convolution .................................................................................... 59
9.4. High pass filter derived with discrete computations ...................................... 60
9.5. Band pass filter derived with discrete computations ....................................... 61
9.6. Band stop filter derived with discrete computations ..................................... 62
10.1. Windowing of a finite impulse response filter ................................................ 63
10.2. Rectangular window ........................................................................................ 63
10.3. Hamming window ............................................................................................ 64
10.4. Bartlett-Hann window ................................................................. 65
10.5. Tukey window ........................................................................... 66
10.6. Kaiser window ........................................................................... 67
10.7. Modified Bessel function of the first kind ........................................ 68
10.10. Kaiser window (simplified) ......................................................... 68
10.13. Windowing of high pass filters ..................................................... 69
12.1. Continuous Fourier transform ....................................................... 76
12.2. Continuous inverse Fourier transform ......................................... 76
12.3. Discrete Fourier transform .......................................................... 76
12.4. Discrete inverse Fourier transform ............................................... 77
12.9. Redundancy of the discrete Fourier transform of real data .......... 78
12.11. Discrete Fourier transform of a simple real-valued wave ............ 78
12.14. Magnitude of the components of DFT of real data .................... 79
12.15. Phase of the components of DFT of real data ......................... 79
12.18. Discrete Fourier transform of a simple complex-valued wave .... 81
12.19. Magnitude of the components of DFT of complex data .......... 81
12.20. Phase of the components of DFT of complex data .................. 81
12.21. Magnitude response of a finite impulse response filter with the DFT 82
12.22. Phase response of a finite impulse response filter with the DFT .... 83
12.31. FIR filter with the inverse DFT .................................................. 85
12.32. Generalized form of the inverse DFT ......................................... 85
12.33. Ideal continuous magnitude response of a low pass filter .......... 88
12.34. Inverse continuous Fourier transform of an ideal low pass magnitude 89
12.35. Low pass filter created with sampling of continuous time computations 89
12.36. High pass filter created with sampling of continuous time computations 90
12.37. Band pass filter created with sampling of continuous time computations 90
12.38. Band stop filter created with sampling of continuous time computations 90
12.41. Hamming family windows ......................................................... 92
12.42. Angular frequencies ................................................................. 92
12.43. Dolph-Chebychev magnitude response ..................................... 93
12.44. Chebychev polynomials ............................................................. 93
12.45. Hyperbolic cosine .................................................................... 93
12.46. Hyperbolic arccosine ................................................................. 93
12.47. Dolph-Chebychev filter ........................................................... 94
12.48. Dolph-Chebychev window ......................................................... 94
12.49. Fourier transform of a product .................................................. 96
12.50. Alternative Dolph-Chebychev window definition ....................... 96
13.3. Phase difference between DFT components and actual frequencies 102
13.7. Limits on the phase difference between DFT components and actual frequencies 104
13.9. Danielson-Lanczos lemma .......................................................... 106
14.3. Coherent gain of a window ........................................................ 109
<table>
<thead>
<tr>
<th>Equation Number</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>14.4</td>
<td>Equivalent noise bandwidth of a filter</td>
<td>110</td>
</tr>
<tr>
<td>14.8</td>
<td>Equivalent noise bandwidth of a window</td>
<td>112</td>
</tr>
<tr>
<td>14.9</td>
<td>Equivalent noise bandwidth of the rectangular window</td>
<td>112</td>
</tr>
<tr>
<td>14.10</td>
<td>Processing gain / loss</td>
<td>113</td>
</tr>
<tr>
<td>14.13</td>
<td>Scalloping loss</td>
<td>114</td>
</tr>
<tr>
<td>14.14</td>
<td>Overlap correlation</td>
<td>118</td>
</tr>
<tr>
<td>15.2</td>
<td>Feedforward comb filter</td>
<td>128</td>
</tr>
<tr>
<td>15.3</td>
<td>Feedback comb filter</td>
<td>129</td>
</tr>
<tr>
<td>15.4</td>
<td>Tapped delay line</td>
<td>132</td>
</tr>
<tr>
<td>15.5</td>
<td>Shroeder all pass filter</td>
<td>133</td>
</tr>
<tr>
<td>15.7</td>
<td>Decay of the Shroeder all pass filter</td>
<td>135</td>
</tr>
<tr>
<td>15.8</td>
<td>Shroeder-Moore low pass feedback comb filter</td>
<td>135</td>
</tr>
<tr>
<td>15.12</td>
<td>Deterministic deconvolution</td>
<td>144</td>
</tr>
<tr>
<td>16.2</td>
<td>Discrete-time Fourier transform</td>
<td>147</td>
</tr>
<tr>
<td>16.3</td>
<td>Inverse discrete-time Fourier transform</td>
<td>147</td>
</tr>
<tr>
<td>16.6</td>
<td>Bilateral Z transform</td>
<td>148</td>
</tr>
<tr>
<td>16.7</td>
<td>Linearity of the Z transform</td>
<td>148</td>
</tr>
<tr>
<td>16.8</td>
<td>Time-shifting of the Z transform</td>
<td>149</td>
</tr>
<tr>
<td>16.11</td>
<td>Transfer function of a system</td>
<td>149</td>
</tr>
<tr>
<td>16.12</td>
<td>Magnitude response with the Z transform</td>
<td>149</td>
</tr>
<tr>
<td>16.13</td>
<td>Phase response with the Z transform</td>
<td>149</td>
</tr>
<tr>
<td>16.16</td>
<td>Transfer function of a feedforward comb filter</td>
<td>150</td>
</tr>
<tr>
<td>16.17</td>
<td>Magnitude response of a feedforward comb filter</td>
<td>150</td>
</tr>
<tr>
<td>16.19</td>
<td>Transfer function of a feedback comb filter</td>
<td>150</td>
</tr>
<tr>
<td>16.20</td>
<td>Magnitude response of a feedback comb filter</td>
<td>151</td>
</tr>
<tr>
<td>16.22</td>
<td>Transfer function of a finite impulse response filter</td>
<td>152</td>
</tr>
<tr>
<td>16.25-31</td>
<td>Gibbs phenomenon with finite impulse response filters</td>
<td>153</td>
</tr>
<tr>
<td>16.36</td>
<td>General form of filter transfer functions</td>
<td>155</td>
</tr>
<tr>
<td>16.37</td>
<td>General from of filter impulse responses</td>
<td>156</td>
</tr>
<tr>
<td>16.38</td>
<td>An infinite impulse response all pass filter</td>
<td>156</td>
</tr>
<tr>
<td>16.40</td>
<td>Transfer function of the Shroeder all pass filter</td>
<td>156</td>
</tr>
<tr>
<td>17.3</td>
<td>Laplace transform</td>
<td>157</td>
</tr>
<tr>
<td>17.4</td>
<td>Inverse Laplace transform</td>
<td>157</td>
</tr>
<tr>
<td>17.5</td>
<td>Relationship between the Laplace transform and the Z transform</td>
<td>158</td>
</tr>
<tr>
<td>17.7</td>
<td>Time-shifting property of the Laplace transform</td>
<td>158</td>
</tr>
<tr>
<td>17.8</td>
<td>Transfer functions with the Laplace transform</td>
<td>158</td>
</tr>
<tr>
<td>17.9</td>
<td>Transfer function of the second order low pass Butterworth filter</td>
<td>159</td>
</tr>
<tr>
<td>17.14</td>
<td>Laplace transform of exponential decay</td>
<td>160</td>
</tr>
<tr>
<td>17.18</td>
<td>Impulse response of the impulse invariant second order low pass Butterworth filter</td>
<td>161</td>
</tr>
<tr>
<td>17.21</td>
<td>Butterworth low pass magnitude response</td>
<td>162</td>
</tr>
<tr>
<td>17.31</td>
<td>Transfer function of the low pass Butterworth filter</td>
<td>166</td>
</tr>
</tbody>
</table>
Table of equations

17.33. Butterworth polynomial ................................................................. 167
18.1-18.2. Bilinear transformation .......................................................... 168
18.3. Derivation of the bilinear transformation ............................................ 168
18.4. Stability of the bilinear transformation ............................................... 169
18.5. Second order low pass Butterworth filter with the bilinear transformation .......................... 169
18.10. Butterworth high pass magnitude response ....................................... 171
18.11.-18.12. Normalized Butterworth transfer function and the Butterworth polynomials ..... 171
18.16. Transfer function of the second order Butterworth band pass filter .............. 173
18.22. Phase response of a second order low pass Butterworth filter ................ 176
18.26. Equivalent noise bandwidth of the first order low pass Butterworth filter ........ 177
18.28. Warping of the frequency domain by the bilinear transformation .............. 178
18.32. Biquad transformation ...................................................................... 179
19.1. Bessel filter and the Bessel polynomials .............................................. 180
19.5. Third order high pass Bessel filter ..................................................... 181
19.9. Second order band stop Bessel filter .................................................. 182
19.10. Transfer function of the Chebychev type I filter ................................. 184
19.13. Second order low pass Chebychev type I filter .................................... 185
19.16. Magnitude response of the low pass Chebychev type I filter .................. 186
19.18. Limits of the magnitude response of the Chebychev type I filter in the pass band .... 186
19.20. Second order high pass Chebychev type I filter .................................... 188
19.22. Transfer function of the Chebychev type II filter .................................. 189
19.28. Fourth order low pass Chebychev type II filter .................................... 191
19.30. Magnitude response of the low pass Chebychev type II filter ................. 192
19.31. Limits of the magnitude response of the Chebychev type II filter in the stop band .... 192
19.39. Adjustment to the gain of odd order Chebychev type II filters .................. 196
19.41. Infinite impulse response notch filter ................................................. 196
19.42. First order notch filter ....................................................................... 197
19.45. Shelving filter .................................................................................. 199
19.47. Second order low-boost shelving filter .............................................. 199
19.48. Second order high-cut shelving filter .................................................. 200
19.50. Second order band-boost shelving filter ............................................. 201
19.51. Second order peak filter .................................................................... 203
20.1. Gain function of the elliptic filter ....................................................... 204
20.2. Second order elliptic rational function .............................................. 205
20.7. Second order elliptic low pass filter .................................................... 206
20.9. Third order elliptic rational function ................................................... 207
20.10. Nesting property of the elliptic rational function ................................... 207
20.17. Fourth order elliptic low pass filter ................................................... 208
20.30-20.35. Algorithm for an equiripple FIR filter ...................................... 214
22.2. Example of noise shaping.................................................................... 227
22.6. First order impulse invariant low pass Butterworth filter .......................................................... 228
23.9. Hilbert transform .......................................................................................................................... 231
23.10. Amplitude envelope .................................................................................................................. 233
24.3. Energy preservation in the Haar wavelet transform ...................................................................... 235
24.4.-24.5. Computing the first level Haar wavelet transform ............................................................... 236
24.6. Reconstructing a signal from the Haar wavelet transform ............................................................. 237
24.9.-24.10. Discrete Haar wavelets ...................................................................................................... 238
24.11. Continuous Haar wavelets .......................................................................................................... 240
24.12. Orthogonality of the Haar wavelets ............................................................................................. 240
24.13. Orthonormality of the Haar wavelets ........................................................................................... 241
24.14.-24.16. Level 1 Daubechies Daub4 wavelet transform ................................................................. 243
24.19. Reconstructing a signal from the Daub4 wavelet transform ..................................................... 245
24.20. Orthogonality of wavelets ............................................................................................................ 247
24.21. Energy preservation with wavelets .............................................................................................. 247
A.1. Bartlett-Hann window .................................................................................................................... 250
A.2. Blackman window ........................................................................................................................ 250
A.3. Blackman-Harris window ............................................................................................................... 250
A.4. Blackman-Nuttall window ............................................................................................................. 250
A.5. Bohman window ............................................................................................................................ 250
A.6. Dolph-Chebychev window ............................................................................................................ 251
A.7. Flat top window ............................................................................................................................ 251
A.8. Gaussian window .......................................................................................................................... 251
A.9. Approximate confined Gaussian window ..................................................................................... 251
A.10. Generalized normal window ......................................................................................................... 251
A.11. Generalized cosine windows ........................................................................................................ 252
A.12. Hamming window ........................................................................................................................ 252
A.13. Hann window ................................................................................................................................ 252
A.14. Hann-Poisson window .................................................................................................................. 252
A.15. Kaiser window ............................................................................................................................. 252
A.16. Kaiser-Bessel window .................................................................................................................. 252
A.17. Lanczos window ............................................................................................................................ 253
A.18. Nuttall window ............................................................................................................................ 253
A.19. Parzen window ............................................................................................................................ 253
A.20. Planck-taper window .................................................................................................................... 253
A.21. Poisson window ............................................................................................................................ 254
A.22. Power-of-cosine windows ............................................................................................................. 254
A.23. Rectangular window ..................................................................................................................... 254
A.24. Sine window ................................................................................................................................. 254
A.25. Triangular window with zero end points ..................................................................................... 254
A.26. Triangular window with non-zero end points ............................................................................. 254
A.27. Tukey window ............................................................................................................................. 254
A.28. Ultraspherical window ................................................................................................................. 255
A.29. Welch window .............................................................................................................................. 255
Index

A

aliasing, 11, 36, 265
  anti-aliasing filter, 36
all pass filter, 58, 156
  finite impulse response, 58
  infinite impulse response, 156
  Shroeder, 132, 156
amplitude
  envelope, 55, 229
  normalized, 20
  of simple wave in signal, 25
  peak. See peak amplitude
  root mean square, 19, 29
amplitude flatness, 118
angular frequency, 92
anti-aliasing filter, 36
attenuation. See stop band attenuation

B

band pass filter, 60
  Butterworth, 173
  FIR (continuous derivation), 90
  FIR (discrete derivation), 60
  Hilbert transform, 232
band stop filter, 60
  Bessel, 182
  Chebychev type II, 193
  FIR (continuous derivation), 90
  FIR (discrete derivation), 61
Bartlett window, 254
Bartlett-Hann window, 65, 72, 250, 260
basic spline window, 253
Bessel filter, 180
  magnitude response, 181
  phase response, 183
  transfer function, 180
Bessel function. See modified Bessel function of the first kind
  bilinear transformation, 168
  and stability, 169
  biquad transformation, 178, 199
Blackman window, 250, 260

exact, 250, 260
  generalized, 260
Blackman-Harris window, 250, 260
Blackman-Nuttall window, 250, 260
Bohman window, 250, 260
B-spline window. See basic spline window
Butterworth filter, 171
  and noise shaping, 228
  impulse invariant, 159
  magnitude response, 161
  maximum flatness, 163
  no ripples, 163
  phase response, 175
  transfer function, 171
Butterworth polynomials, 167
  normalized, 171

Chebychev polynomials, 93, 186
Chebychev type I filter, 184
  magnitude response, 185
  transfer function, 184
Chebychev type II filter, 189
  magnitude response, 191
  transfer function, 189
coherent gain, 109, 260
comb filter, 128
  feedback, 129
  feedback transfer function, 150
  feedforward, 128
  feedforward transfer function, 150
  Shroeder, 134
  Shroeder-Moore, 135
compression
  of data, 234
  of dynamics, 229
continuous Fourier transform, 76
  sampling of transform, 88
convolution, 37
  with simple wave, 39
convolution theorem, 91
cross-fading, 103
cubic soft clipper, 31
cutoff frequency, 39
DSP for Audio Applications: Formulae

Index

D

Danielson-Lanczos lemma, 105
Daubechies Daub4 wavelet transform, 243
db. See decibel
decibel, 20
  unloaded, 22
  voltage, 22
de-convolution, 136
delay, 14
  tapped delay line, 132
  transfer function, 150
DFT. See discrete Fourier transform
digital signal processing, 1
Dirac comb, 157
Dirac delta function, 49
Dirichlet kernel, 41
discrete Fourier transform, 76
  and FIR filters, 81
  and Hamming windows, 90
  and pitch shifting, 100
  generalized inverse, 85
  of complex valued data, 81
  of real valued data, 79
  redundancy, 80
discrete-time Fourier transform, 147
distortion, 30
dithering, 225
Dolph-Chebychev, 92
  filter, 94
  magnitude response, 92
  window, 94, 251, 260
downsampling, 35
DSP. See digital signal processing

E

echo, 130
  transfer function, 150
e elliptic filter, 204
equal tempered scale, 75
equalizer, 72
equiripple filter, 204
equivalent noise bandwidth, 110, 176, 260
Euler's formula, 77
even order harmonics, 33

echo, 130
  transfer function, 150
e elliptic filter, 204
equal tempered scale, 75
equalizer, 72
equiripple filter, 204
equivalent noise bandwidth, 110, 176, 260
Euler's formula, 77
even order harmonics, 33

F

fast Fourier transform, 105
filter
  all pass. See all pass filter
  band pass. See band pass filter
  band stop. See band stop filter
  combining, 72, 264
  equiripple. See equiripple filter
  feedback, 51
  feedforward, 51
  finite impulse response. See finite impulse response
general form, 52
  high pass. See high pass filter
  IIR transformations, 171
  impulse invariant, 169
  infinite impulse response. See infinite impulse response
  low pass. See low pass filter
  notch. See notch filter
  optimized, 216
  finite impulse response, 50
FIR. See finite impulse response
  flat top window, 251, 259
  Flat top window, 261
  formant, 13
  Fourier analysis, 23
  Fourier series, 26
  Fourier transform. See continuous Fourier transform
  or discrete Fourier transform
frequency, 5
  angular. See angular frequency
cutoff. See cutoff frequency
  fundamental. See fundamental frequency
  normalized. See normalized frequency
transition. See cutoff frequency
frequency content, 25
frequency domain analysis, 107
frequency response, 19, 45
fundamental frequency, 13

G

Gaussian window, 251, 261
  approximate confined, 251, 261
Gegenbauer polynomials, 255
generalized cosine window, 252
generalized normal window, 251, 261
Gibbs phenomenon, 46, 74, 90, 154

H
Haar wavelet transform, 236
Hamming window, 63, 252, 261
family, 90
Hann window, 108, 252, 261
Hann-Poisson window, 252, 261
hard clip, 30
harmonic distortion, 31
harmonics, 13, 31
Hertz, 7
high pass filter, 57, 59
  Bessel, 180
  Butterworth, 172
  Chebychev type I, 187
FIR (continuous derivation), 89
FIR (discrete derivation), 60
highest sidelobe level, 121, 260
Hilbert transform, 229

I
IIR. See infinite impulse response
impulse, 49
impulse response, 49, 52
  and transfer functions, 155
  finite. See finite impulse response
infinite. See infinite impulse response
of a FIR filter. See finite impulse response
of an IIR filter. See infinite impulse response
of comb filter, 128
of reverb, 136
  of Shroeder all pass filter, 133
  of Shroeder-Moore filter, 135
inharmonic overtone, 13
initial phase, 2
inversion. See spectral inversion of filter
inverted phase / polarity, 3

J
just tempered scale, 75

K
Kaiser window, 67, 252, 261
Kaiser-Bessel window, 252, 262
Kotelnikov. See Nyquist-Shannon sampling theorem
Kronecker delta function, 49

L
Lanczos window, 252, 262
Laplace transform, 157
  and stability, 165
  and transfer functions, 158
leaky integrator, 129
L'Hopital's rule, 41
linear phase, 54
linear time-invariant system, 49
low pass filter, 18
  Butterworth, 159
  Chebychev type I, 184
  Chebychev type II, 190
  FIR (continuous derivation), 89
  FIR (discrete derivation), 42
  ideal, 39

M
magnitude response, 19, 45
  and the DFT, 79, 81
  and the Laplace transform, 159
  and the Z transform, 149
  of Bessel filter, 181
  of Butterworth filter, 161
  of Chebychev type I filter, 185
  of Chebychev type II filter, 191
  of comb filter, 128, 130
  of FIR filter, 45
  of Hilbert transform, 232
  of notch filter, 197
  of optimized filter, 220
  of running average filter, 20
memoryless operation, 34
mixing, 14
modified Bessel function of the first kind, 68
multitap delay, 132

N
noise gate, 229
noise shaping, 227
normalized frequency, 9
notch filter, 196
  magnitude response, 197
<table>
<thead>
<tr>
<th>Index</th>
<th>DSP for Audio Applications: Formulae</th>
</tr>
</thead>
<tbody>
<tr>
<td>transfer function, 196</td>
<td>rectangular window, 63, 254, 257, 262</td>
</tr>
<tr>
<td>Nuttall window, 253, 262</td>
<td>reverberation, 130</td>
</tr>
<tr>
<td>Nyquist-Shannon sampling theorem, 10</td>
<td>impulse reverb, 136</td>
</tr>
<tr>
<td>and distortion, 34</td>
<td>Shroeder, 132</td>
</tr>
<tr>
<td>odd order harmonics, 30</td>
<td>ripples</td>
</tr>
<tr>
<td>optimization, 217</td>
<td>and optimization, 216</td>
</tr>
<tr>
<td>orthogonal simple waves, 23</td>
<td>and the Gibbs phenomenon. See Gibbs</td>
</tr>
<tr>
<td>overlap correlation, 118, 260</td>
<td>phenomenon</td>
</tr>
<tr>
<td>overtone, 13</td>
<td>maximum, 154</td>
</tr>
<tr>
<td>padding, 59</td>
<td>of FIR filter, 45</td>
</tr>
<tr>
<td>partial harmonics, 13</td>
<td>of running average filter, 18</td>
</tr>
<tr>
<td>partial wave, 13</td>
<td>RMS. See root mean square</td>
</tr>
<tr>
<td>Parzen window, 253, 262</td>
<td>root mean square, 19</td>
</tr>
<tr>
<td>pass band, 39</td>
<td>running average filter, 14</td>
</tr>
<tr>
<td>PCM. See pulse code modulation</td>
<td></td>
</tr>
<tr>
<td>peak amplitude, 4</td>
<td></td>
</tr>
<tr>
<td>peak filter, 203</td>
<td></td>
</tr>
<tr>
<td>phase, 2</td>
<td></td>
</tr>
<tr>
<td>as portion of cycle, 7</td>
<td></td>
</tr>
<tr>
<td>initial. See initial phase</td>
<td></td>
</tr>
<tr>
<td>inverted. See inverted phase / polarity</td>
<td></td>
</tr>
<tr>
<td>phase response, 52</td>
<td></td>
</tr>
<tr>
<td>and the DFT, 79, 81</td>
<td></td>
</tr>
<tr>
<td>and the Laplace transform, 159</td>
<td></td>
</tr>
<tr>
<td>and the Z transform, 149</td>
<td></td>
</tr>
<tr>
<td>of Bessel filter, 183</td>
<td></td>
</tr>
<tr>
<td>of Butterworth filter, 175</td>
<td></td>
</tr>
<tr>
<td>of symmetric FIR filters, 53, 84</td>
<td></td>
</tr>
<tr>
<td>pitch shifting, 100</td>
<td></td>
</tr>
<tr>
<td>Planck-taper window, 253, 262</td>
<td></td>
</tr>
<tr>
<td>Poisson window, 254, 262</td>
<td></td>
</tr>
<tr>
<td>power-of-cosine window, 254</td>
<td></td>
</tr>
<tr>
<td>processing loss, 113, 260</td>
<td></td>
</tr>
<tr>
<td>pulse code modulation, 8</td>
<td></td>
</tr>
<tr>
<td>Pythagorean tempered scale, 75</td>
<td></td>
</tr>
<tr>
<td>quantization, 223</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>radian, 92</td>
<td></td>
</tr>
</tbody>
</table>
and the bilinear transformation, 169
and the Laplace transform, 165
and the Z transform, 166
stop band, 39
stop band attenuation, 46

tapering function, 63
tapped delay line, 132
timbre, 13
transfer function, 149
and impulse response, 155
and magnitude response, 149
and phase response, 149
and the Laplace transform, 158
and the Z transform, 149
of all pass filter, 156
of Bessel filter, 180
of Butterworth filter, 171
of Chebychev type I filter, 184
of Chebychev type II filter, 189
of feedback comb filter, 150
of feedforward comb filter, 150
of FIR filter, 152
of IIR filter, 155
of notch filter, 196
of Shroeder all pass filter, 156
transition band, 45
transition frequency. See cutoff frequency
triangular window, 254, 262
Tukey window, 66, 254, 262

ultraspherical polynomials, 255
ultraspherical window, 255, 263
undertone, 13

upsampling, 35

wavelet transform
  compacting energy, 249
  Daubechies Daub4, 243
  Haar, 236
Welch window, 255, 263
window, 63
  amplitude flatness, 118
  coherent gain, 109
  equivalent noise bandwidth, 110
  highest sidelobe level, 121
  measures, 107, 260
  of band pass / band stop filter, 70
  of high pass filter, 69
  of low pass filter, 63
  overlap correlation, 118
  performance, 123
  processing loss, 113
  scalloping loss, 114
  sidelobe falloff, 117
  worst case processing loss, 116
wolf interval, 75
worst case processing loss, 116, 260

z- complex plane, 166
Z transform, 148
  and stability, 166
  and the discrete-time Fourier transform, 147
  and the Laplace transform, 158